# SPECIALLY STRUCTURED THREE STAGES FLOW SHOP SCHEDULING PROCESSING TIME ASSOCIATED WITH PROBABILITIES INCLUDING TRANSPORTATION TIME TO MINIMIZE THE RENTAL COST

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## Abstract

This paper deals with a heuristic algorithm to minimize the rental cost of machines for specially structured three stage flow shop scheduling problem under specified rental policy in which processing times are associated with their respective probabilities. Further, the transportation time from one machine to other machine is being considered. A numerical illustration is given to clarify the algorithm.

***Keywords***: Specially structured flow shop scheduling, rental policy, processing time, utilization time, transportation time.

## 1. Introduction:

Scheduling is one of the optimization problem found in real industrial content for which several heuristic procedures have been successfully applied. Flow shop is the classical and most studied manufacturing environment in scheduling literature, only some efforts had been made to minimize the rental cost of machines minimization of makespan does not always lead to minimum rental cost. Further, in most of the literature the processing time of times is considered to be random with a goal to minimize the makespan and mean time flow. But there are significant situations in which the processing times are not random, but follow a some well defined structural conditions. Johnson (1954) gave procedure for finding the optimal schedule for n-job, two machines flow shop problem with minimization of makespan (i.e. total elapsed time) as an objective, Gupta J.N.D. (1975) gave an algorithm to find the optimal schedule for specially structured flow shop scheduling. Smith et at (1975) considered a special case in which the job processing times on the first or last machine are the longest and showed that the problem can be solved in polynomial time. The work was developed by Ignall and Scharge (1965), Bagga (1969), Maggu and Dass (2005), Szwarch (1977), Yoshida and Hitomi (1979), Singh T.P. (1985), Chandrasekhran (1992), Gupta Deepak (2005), Narain (2005) etc by considering the various parameters. Gupta, Sharma & Bala (2012) studied specially structured two stage flow shop problem to minimize the rental cost of the machines under pre defined rental policy in which the probabilities have been associated with processing time. The present paper is an attempt to develop a heuristic algorithm to minimize the renal cost for three stages specially structured flow shop scheduling under a specified renal policy including transportation time. The problem discussed here is wider and has significant use of theoretical results in process industries.

## 2. Practical Situation:

Many applied and experimental situations exist in our day to day working in factories and industrial production concerns etc. In many manufacturing companies different jobs are processed on various machines. These jobs are required to process in a machine shop A, B,

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C…. in a specified order. When the machines on which jobs are to be processed are placed at different places, the transportation time (which includes loading time, moving time and unloading time etc) has a significant role in production concern.

**3. Notations:**

| | |
|---|---|
| S: | Sequence of jobs 1,2,3,….,n |
| $S_k$: | Sequence obtained by applying Johnson's procedure k= 1, 2, 3, ….. |
| $M_j$: | Machine j, j= 1,2, 3. |
| $a_{ij}$: | Processing time of $i^{th}$ job on machine $M_j$ |
| $p_{ij}$: | Probability associated to the processing time $a_{ij}$. |
| $A_{ij}$: | Expected processing time of $i^{th}$ job of sequence $S_k$ on machine $M_j$. |
| $T_{i,j\to k}$: | Transportation time of $i^{th}$ job from $i^{th}$ machine to $k^{th}$ machine. |
| $t_{ij}(S_k)$: | completion time of $i^{th}$ job of sequence $S_k$ on machine $M_j$. |
| $T_{ij}(S_k)$: | Idle time of machine $M_j$ for job i in the sequence $S_k$. |
| $U_j(S_k)$: | Utilization time for which machine $M_j$ is required. |
| $R(S_k)$: | Total rental cost for the sequence $S_k$ of all machine. |
| $C_j$: | Rental cost of $j^{th}$ machine. |
| $CT(S_k)$: | Total completion time of the jobs for sequence $S_k$. |

**4. Assumptions:**

1. Machines break down is not considered. This simplifies the problem by ignoring the stochastic component of the problem.
2. Jobs are independent to each other.
3. Pre- emption is not allowed i.e. once a job started on a machine the process on machine can't be stopped unless the job in completed.
4. Either $A_{j2} \leq A_{i1}$ or $A_{i2} \leq A_{j3}$ $\forall$ i, j i.e. processing time on $1^{st}$ machine will always be longer then the processing time $2^{nd}$ machine of all jobs or Processing time on $3^{rd}$ machine will always be greater or equal then the processing time of $2^{nd}$ machine of all the jobs.

**4.1 Definition:**

Completion time of $i^{th}$ job on machine $M_j$ is denoted by $t_{ij}$ and is defined as:

$t_{ij}=$ $\max(t_{i-1,j}, t_{i,j-1} + T_{i,j-1\to j}) + a_{ij} \times p_{ij}$ , $J \geq 2$.

$=$ $\max(t_{i-1,j}, t_{i,j-1} + T_{i,j-1\to j}) + A_{ij}$where

Aij is expected processing time of $i^{th}$ job on $j^{th}$ machine.

**5. Rental Policy:**

The machine will be taken on rent as and when they are required and are returned as and when they are no longer required.

**6. Problem Formulation:**

Let some job (i= 1, 2, … n) are to be processed on three machines $M_j$ (j= 1, 2, 3) under the specified rental policy P. let $a_{ij}$ be the processing time of $i^{th}$ job on $j^{th}$ machine with probabilities $p_{ij}$. Such that $\sum p_{ij} =1$. Let $a_{ij}$ be the expected processing time of $i^{th}$ job on $j^{th}$ machine such that either $A_{i1} \geq A_{i2}$ $\forall$ i, j or $A_{i3} \geq A_{i2}$ $\forall$ i, j. Our aim is to find the sequence (Sk) of the jobs which minimize the rental cost of the machines.

Table 1. The mathematical model of the problem in matrix form.

| Jobs | Machine $M_1$ | | $T_{i1\to 2}$ | Machine $M_2$ | | $T_{i2\to 3}$ | Machine $M_3$ | |
|---|---|---|---|---|---|---|---|---|
| i | $a_{i1}$ | $p_{i1}$ | $t_i$ | $a_{i2}$ | $p_{i2}$ | $g_i$ | $a_{i3}$ | $p_{i3}$ |
| 1 | $a_{11}$ | $p_{11}$ | $t_1$ | $a_{12}$ | $p_{12}$ | $g_1$ | $a_{13}$ | $p_{13}$ |
| 2 | $a_{21}$ | $p_{21}$ | $t_2$ | $a_{22}$ | $p_{22}$ | $g_2$ | $a_{23}$ | $p_{23}$ |
| 3 | $a_{31}$ | $p_{31}$ | $t_3$ | $a_{32}$ | $p_{32}$ | $g_3$ | $a_{32}$ | $p_{33}$ |
| . | . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . | . |
| n | $a_{n1}$ | $p_{n1}$ | $t_n$ | $a_{n2}$ | $p_{n2}$ | $g_n$ | $a_{n3}$ | $P_{n3}$ |

Mathematically, the problem is stated as :

Minimize $R(S_k) = \sum A_{i1} \times C_1 + U_2 (S_k) \times C_2 + U_3 (S_k) \times C_3$

Subject to constraint: Rental Policy (P)

i.e. our objective is to minimize rental cost of machines while minimizing the utilization time.

**Algorithm:**

Step 1: Calculate the expected processing times:
$A_{ij} = a_{ij} \times p_{ij}\ \forall$ i, j

Step 2: Check the condition : either $A_{j2} \leq A_{i1}$
Or $A_{i2} \leq A_{j3}\ \forall$ i, j
i.e. either min $\{A_{i1}\} \geq$ max $\{A_{j2}\}$
or min $\{A_{j3}\} \geq$ max $\{A_{i2}\}\ \forall$ i, j
If the conditions are satisfies then go to step 3, else the data is not in standard form.

Step 3: Introduce the two fictitious machines G and H with processing times $G_i$ and $H_i$ as:
$G_i = A_{i1} + t_i + A_{i2} + g_i$
$H_i = t_i + A_{i2} + A_{i3} + g_i\ \forall$ i

Step 4: Obtain the sequence $S_1$ (say) by applying Johnson's (1954) algorithm on machines G and H.

Step 5: Obtain other sequence by putting $2^{nd}$, $3^{rd}$, ….. $n^{th}$ job of sequence $S_1$ in the $1^{st}$ position and all other jobs of $S_1$ in the same order. Let the sequences be $S_2$, $S_3$, ….., $S_{n-1}$.

Step 6: Compute $\sum A_{i1}$, $U_2(S_k)$, $U_3(S_k)$ and $R(S_k) = \sum A_{i1} \times C_1 + U_2 (S_k) \times C_2 + U_3 (S_k) \times C_3$ for all the possible sequences $S_k$ (k = 1, 2, …., n)

Step 7: Find min $\{R(S_k)\}$; k= 1, 2, … , n le it be minimum for the sequence $S_p$, then the sequence $S_p$ will be the optimal sequence with rental cost $R(S_p)$.

**Numerical Illustration:**

Consider 5 jobs 3 machines flow shop problem with processing time associated with probabilities including transportation time as given in table. The rental cost per unit time for machines $M_1$, $M_2$ and $M_3$ are 4 units, 6 units & 8 units respectively under the rental policy P. our objective in to obtain an optimal schedule for to minimize the total rental cost of minimize the total rental cost of the machines.

Table 2. The processing times with probabilities including transportation time:

| Jobs | Machine $M_1$ | | $T_{i1\to 2}$ | Machine $M_2$ | | $T_{i2\to 3}$ | Machine $M_3$ | |
|---|---|---|---|---|---|---|---|---|
| i | $a_{i1}$ | $p_{i1}$ | $t_i$ | $a_{i2}$ | $p_{i2}$ | $g_i$ | $a_{i3}$ | $p_{i3}$ |
| 1 | 55 | 0.1 | 2 | 20 | 0.2 | 2 | 30 | 0.2 |
| 2 | 30 | 0.2 | 3 | 11 | 0.4 | 2 | 45 | 0.1 |
| 3 | 25 | 0.3 | 2 | 26 | 0.1 | 3 | 30 | 0.2 |
| 4 | 29 | 0.2 | 3 | 15 | 0.2 | 4 | 25 | 0.3 |
| 5 | 25 | 0.2 | 4 | 35 | 0.1 | 2 | 26 | 0.2 |

Solution: The expected processing times $A_{i1}$, $A_{i2}$ and $A_{i3}$ for machines $M_1$, $M_2$ and $M_3$ are with transportation time $t_i$ & $g_i$ from machine $M_1$ to $M_2$ and from $M_2$ to $M_3$ resp. are:

Table 3.

| Jobs | $A_{i1}$ | $T_i$ | $A_{i2}$ | $g_i$ | $A_{i3}$ |
|---|---|---|---|---|---|
| 1 | 5.5 | 2 | 4.0 | 2 | 6.0 |
| 2 | 6.0 | 3 | 4.4 | 2 | 4.5 |
| 3 | 7.5 | 2 | 2.6 | 3 | 6.0 |
| 4 | 5.8 | 3 | 3.0 | 4 | 7.5 |
| 5 | 5.0 | 4 | 3.5 | 2 | 5.2 |

The fictions machines with processing times G1 & Hi are:

Table 4. The processing time for two fictions machines G & H

| Jobs<br>i | Machine G<br>$G_i$ | Machine H<br>$H_i$ |
|---|---|---|
| 1 | 13.5 | 14.0 |
| 2 | 15.4 | 13.9 |
| 3 | 15.1 | 13.6 |
| 4 | 15.8 | 17.5 |
| 5 | 14.5 | 14.7 |

Here each $m_{i1}'' \le m_{j2}''$ $\forall$ i, j
Therefore max ($m_{i1}''$) = 7.65 which is for job 4 is $J_1 = 4$.
And min ($m_{i2}''$) = 7.75 which is for job 3 i.e. $J_n = 3$.

Since $J_1 \neq J_n$ $\because$ we put J1 on the 1st position and Jn on the last position. As per step 9 we get optimal sequences as : $S_1$: 4 – 1 – 2 – 5 – 3 ; $S_2$ : 4 – 1 – 5 – 2 – 3; $S_3$ : 4 – 2 – 1 – 5 – 3; $S_4$: 4 – 2 – 5 – 1 – 3. $S_5$: 4 – 5 – 2 – 1 – 3; $S_6$ : 4 – 5 – 1 – 2 – 3.

Total elapsed time will be same for all the sequence $S_1$, $S_2$, …., $S_6$. Therefore we find in-out table for any of these sequences $S_1$, $S_2$, …, $S_6$; say for $S_1$ : 4 – 1 – 2 – 5 – 3.
In-out table for $S_1$:

Table 5. In-out table for $S_1$:

| Jobs | Machine $M_1$ in-out | Machine $M_2$ in-out |
|---|---|---|
| 4 | 0 – 46 | 46 – 88 |
| 1 | 46 – 61 | 88 – 118 |
| 2 | 61 – 81 | 118 – 153 |
| 5 | 81 – 121 | 153 – 203 |
| 3 | 121 – 151 | 203 – 230 |

Using Johnson (1954) procedure, the sequence with minimum makespan is:
$S_1$: 1 – 5 – 4 – 2 – 3

Other feasible sequences which may correspond to minimum rental cost are:
$S_2$: 5 – 1 – 4 – 2 – 3
$S_3$: 4 – 1 – 5 – 2 – 3,
$S_4$: 2 – 1 – 5 – 4 – 3
$S_5$: 3 – 1 – 5 – 4 – 2

From in-out tables for these sequences, we have

For $S_1$: $CT(S_1) = 44.3$, $U_2(S_1) = 26.9$, $U_3(S_1) = 30.8$, $R(S_1) = 527.0$
For $S_2$: $CT(S_2) = 44.3$, $U_2(S_2) = 25.4$, $U_3(S_2) = 29.8$, $R(S_2) = 510.0$
For $S_3$: $CT(S_3) = 45.0$, $U_2(S_3) = 25.6$, $U_3(S_3) = 30.8$, $R(S_3) = 506.4$
For $S_4$: $ST(S_4) = 45.8$, $U_2(S_4) = 25.4$, $U_3(S_4) = 29.2$, $R(S_4) = 518.0$
For $S_5$: $ST(S_5) = 45.8$, $U_2(S_5) = 27.7$, $U_3(S_5) = 30.7$, $R(S_5) = 531.0$
Therefore, Min $R(S_k) = 506.4$ units and is for the sequence $S_3$.

Hence the sequence $S_3$: 4 – 1 – 5 – 2 – 3 is optional sequence with minimum rental cost 506.4 units although the total elapsed time for $S_3$ is not minimum.

**7. Conclusion:**

The algorithm proposed in this paper for specially structured three stage flow shop scheduling problem to minimize the rental cost of the machines gives an optimal sequence having minimum rental cost of machines irrespective of total elapsed time. The algorithm proposed by Johnson (1954) to find an optimal sequence to minimize the makespan / total elapsed time is not always corresponds to minimum rental cost of the machines. Hence proposed algorithm is more efficient to minimize the rental cost of machines under a specified rental policy.

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